

ACT Math  
Diagnostic  
Test #2

Art Cockerham

# ACT Math Diagnostic Test #2

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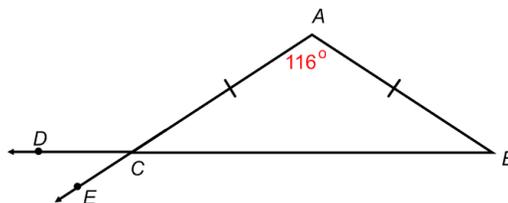
1. Charley's Coffee Shop sells a 20-ounce coffee for \$2.70, while Joe's Java Joint sells a 16-ounce coffee for \$1.92. Which shop's price per ounce is cheaper, and what is that price?

- A. Charley's at \$0.135 per ounce
- B. Charley's at \$0.14 per ounce
- C. Joe's at \$0.11 per ounce
- D. Joe's at \$0.12 per ounce
- E. Joe's at \$0.13 per ounce

2. What is the value of  $x$  that satisfies the equation  $3(x + 1) = 7x - 8$ ?

- F.  $-5/4$
- G.  $-1$
- H.  $5/4$
- J.  $2$
- K.  $11/4$

3. In the figure below,  $B$ ,  $C$  and  $D$  are on the same line. Additionally,  $A$ ,  $C$  and  $E$  are also on the same line. If triangle  $ABC$  is isosceles and the measure of  $\angle CAB$  is 116 degrees, what is the measure of  $\angle DCE$ ?



- A.  $16^\circ$
- B.  $24^\circ$
- C.  $32^\circ$
- D.  $64^\circ$
- E.  $116^\circ$

4. Nick brought 5 shirts, 6 pairs of pants and 3 sweaters with him on vacation. How many different outfits, each made up of 1 shirt, 1 pair of pants and 1 sweater, can he make with the clothes that he brought?

- F. 14
- G. 30
- H. 45
- J. 90
- K. 120

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5. Point  $M$  is the midpoint of segment  $AB$ . If the coordinates of Point  $A$  are  $(-2, 4)$ , the coordinates of Point  $M$  are  $(6, 14)$ , and the coordinates of Point  $B$  are  $(a, b)$ , what is the value of  $a$ ?

- A. 2
- B. 4
- C. 8
- D. 14
- E. 24

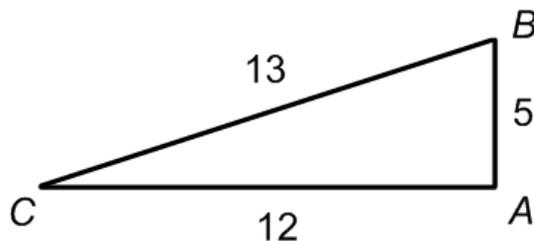
6. If  $x + y = 10$  and  $y = 2x + 1$ , then  $y = ?$

- F. 2
- G. 3
- H. 6
- J. 7
- K. 8

7. A deck of cards contains 15 cards with a black square on them, 15 cards with a blue circle on them and 15 cards with a yellow triangle on them. If Joanna has drawn five cards and is holding 2 black squares and 3 yellow triangles, what is the probability she will draw a card with a blue circle on the next card?

- A.  $1/3$
- B.  $3/10$
- C.  $13/40$
- D.  $3/8$
- E.  $1/2$

8. In the right triangle  $ABC$  (with  $\angle A$  the right angle) shown below, what is  $\cos B$ ?

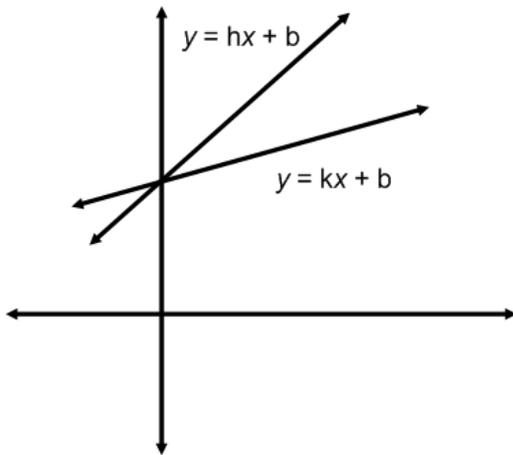


- F.  $5/13$
- G.  $5/12$
- H.  $12/13$
- J.  $12/5$
- K.  $13/5$

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9. The two lines shown in the standard  $(x, y)$  coordinate plane below have the same  $y$ -intercept. What is the relationship between  $h$  and  $k$ , the slopes of the two lines?



- A.  $h = k$
- B.  $h < k$
- C.  $h > k$
- D.  $h < 0$  and  $k > 0$
- E.  $h > 0$  and  $k < 0$

10. Given  $f(x) = 2x - 3$  and  $g(x) = x^2 + 3$ , which of the following is an expression for  $g(f(x))$ ?

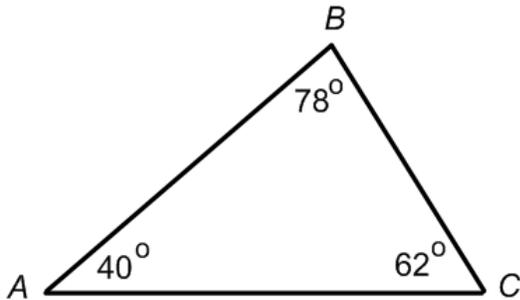
- F.  $4x^2$
- G.  $4x^2 + 6$
- H.  $4x^2 + 12$
- J.  $4x^2 - 6x + 12$
- K.  $4x^2 - 12x + 12$

11. If  $x = y + 5$ , then  $(y - x)^3 = ?$

- A. -125
- B. -25
- C. 5
- D. 25
- E. 125

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12. Given the triangle below, which of the following inequalities must be true?

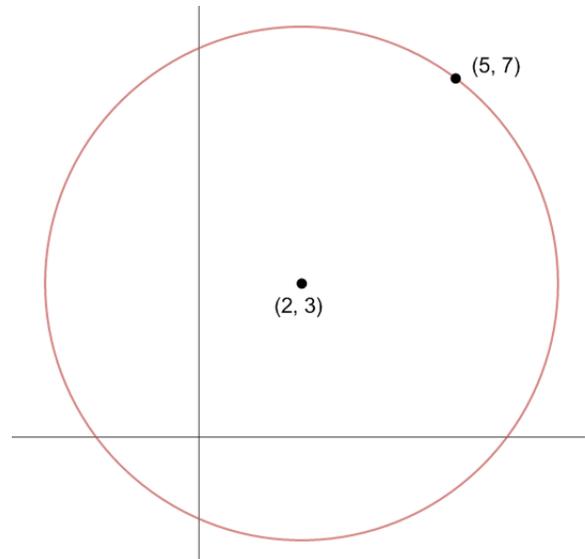


- F.  $AB < AC < BC$
- G.  $AC < AB < BC$
- H.  $AC < BC < AB$
- J.  $BC < AB < AC$
- K.  $BC < AC < AB$

13. The *only* solution to the equation  $x^2 + bx + c = 0$  is  $x = -3$ . What is the value of  $b$ ?

- A. -9
- B. -6
- C. 3
- D. 6
- E. 9

14. The circle below has its center at  $(2, 3)$  and contains the point  $(5, 7)$ . What is the area of the circle?



- F.  $16\pi$
- G.  $25\pi$
- H.  $49\pi$
- J.  $64\pi$
- K.  $100\pi$

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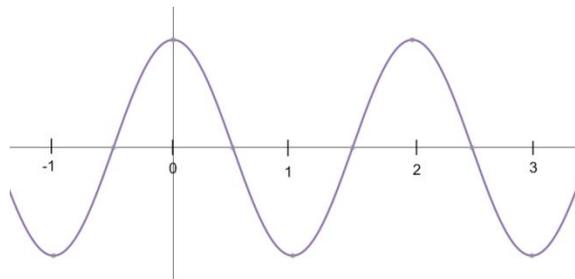
15. The speed of sound in water is approximately  $1.49 \times 10^3$  meters per second. About how far, in meters, would a sound wave in water travel in 5 minutes?

- A.  $8.94 \times 10^4$
- B.  $1.32 \times 10^5$
- C.  $4.47 \times 10^5$
- D.  $7.35 \times 10^6$
- E.  $2.68 \times 10^7$

16. Brendan has a total of 6 tests in his U.S. History class this semester. On the first 3 tests his average was 89 and on the next 2 tests his average was 87. What does he need to get on the 6<sup>th</sup> test for his average to be exactly 90?

- F. 90
- G. 93
- H. 94
- J. 97
- K. 99

17. The *period* of a function is the smallest value of  $p$  such that  $f(x) = f(x + p)$  for all values of  $x$ . The graph shown below is that of the function  $f(x) = 3\cos(\pi x)$ . What is the period of the function?



- A. 1
- B. 2
- C. 3
- D.  $\pi$
- E.  $2\pi$

18. In an arithmetic sequence, each term after the first is found by adding the same number to the preceding term. If the sum of the first six terms of an arithmetic sequence is 78 and the sixth term is 23, what is the second term?

- F. 3
- G. 4
- H. 7
- J. 10
- K. 19

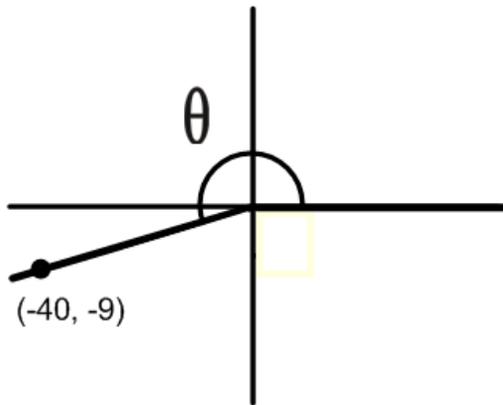
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19. For all values of  $\theta$ ,  $(3\cos\theta - 3\sin\theta)^2 = ?$

- A.  $3\cos^2\theta - 3\sin^2\theta$
- B.  $9\cos^2\theta - 9\sin^2\theta$
- C.  $9\cos^2\theta - 9\cos\theta\sin\theta + 9\sin^2\theta$
- D.  $9 - 18\cos\theta\sin\theta$
- E.  $18\cos\theta\sin\theta$

20. In the standard  $(x, y)$  coordinate plane below, an angle is shown whose vertex is the origin. One side of this angle  $\theta$  passes through the point  $(-40, -9)$  and the other side of the angle is on the positive  $x$ -axis. What is  $\sin\theta$ ?



- F.  $-9/40$
- G.  $-9/41$
- H.  $-40/41$
- J.  $9/41$
- K.  $40/41$

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## Answer Key

Answer	EA	AG	GT	Topic(s)
1. D	X			Rates & Ratios
2. K	X			Solving Equations
3. C			X	Angles; Triangles
4. J	X			Counting Principle
5. D		X		Midpoint Formula
6. J		X		Systems of Equations
7. D	X			Probability
8. F			X	Right Triangle Trigonometry
9. C		X		Graphs of Linear Equations; Slope
10. K		X		Composition of Functions
11. A	X			Equations & Expressions
12. J			X	Properties of Triangles
13. D	X			Polynomial Equations; Factors
14. G		X		Distance Formula; Area of a Circle
15. C	X			Rates; Scientific Notation
16. K	X			Averages
17. B			X	Period of a Function
18. H		X		Sequences
19. D			X	Algebraic Expressions; Trigonometry Identities
20. G			X	Right Triangle Trigonometry (SOH-CAH-TOA)

EA = Pre-Algebra & Elementary Algebra

AG = Intermediate Algebra & Coordinate Geometry

GT = Plane Geometry & Trigonometry

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## Solutions

1. D

For each coffee shop, find the unit rate (the cost per ounce).

Charley's:  $2.70/20 = 0.135$  or 13.5 cents/oz

Joe's:  $1.95/16 = 0.12$  or 12 cents/oz

Joe's has a lower cost per ounce.

2. K

Solve the equation using the Distributive Property and opposite operations. Be careful with signs.

$$3(x + 1) = 7x - 8$$

$$3x + 3 = 7x - 8$$

$$3x - 3x + 3 = 7x - 3x - 8$$

$$3 = 4x - 8$$

$$3 + 8 = 4x - 8 + 8$$

$$11 = 4x$$

$$11/4 = 4x/4$$

$$11/4 = x$$

3. C

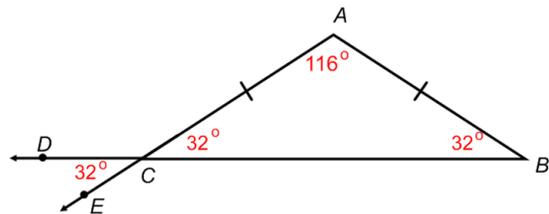
Because triangle  $ABC$  is isosceles, the two base angles have the same measure. Let's call that measure  $x$ .

$$x + x + 116 = 180$$

$$2x = 64$$

$$x = 32$$

Now you can label the diagram with the angles you know.



Angles  $DCE$  and  $ACB$  are vertical angles, so they have the same measure.

4. J

This problem uses the Counting Principle. In order to find out how many different outfits Nick can make, you simply multiply the number of shirts by the number of pairs of pants by the number of sweaters.

$$5 \times 6 \times 3 = 90$$

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5. D

You can look at this problem in a couple of ways. You should concentrate on the  $x$ -coordinates of the points because we are asked to find the  $x$ -coordinate of Point  $B$ . The  $x$ -coordinate of the midpoint is 8 units greater than that of Point  $A$ . Therefore, the  $x$ -coordinate of Point  $B$  must also be 8 more than that of the midpoint but in the other direction. Adding 8 to 6 gets you 14.

You also can do this problem using the Midpoint Formula.

$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\frac{-2 + b}{2} = 6$$

$$-2 + b = 12$$

$$-2 + 2 + b = 12 + 2$$

$$b = 14$$

6. J

You can solve this system of equations using substitution.

$$x + y = 10$$

$$x + 2x + 1 = 10$$

$$3x + 1 = 10$$

$$3x + 1 - 1 = 10 - 1$$

$$3x = 9$$

$$x = 3$$

Now that you know the value of  $x$ , you can substitute again to find  $y$ .

$$y = 2x + 1$$

$$y = 2(3) + 1$$

$$y = 6 + 1$$

$$y = 7$$

7. D

The deck originally had 45 cards in it, 15 each of the three different types of cards. Five cards have been removed from the deck, so it now has 40 cards remaining. Because all of the cards with blue circles are still in the deck, the probability of getting a blue circle on the next card is  $15/40 = 3/8$ .

8. F

The cosine of an angle is the ratio of the length of the adjacent side to the length of the hypotenuse. Therefore,  $\cos B = 5/13$ .

9. C

The slope of a line is defined as the ratio of the change in  $y$  to the change in  $x$ . Both of the lines in the problem have positive slopes, so the line that is steeper will have the greater slope. Therefore,  $h > k$ .

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10. K

In this composition of functions problems, you should replace the  $x$  in the  $g(x)$  with the expression that defines  $f(x)$ .

$$\begin{aligned}g(f(x)) &= (2x - 3)^2 + 3 \\g(f(x)) &= (2x - 3)(2x - 3) + 3 \\g(f(x)) &= 4x^2 - 6x - 6x + 9 + 3 \\g(f(x)) &= 4x^2 - 12x + 12\end{aligned}$$

11. A

You can start by using the first equation to find a value for  $y - x$ .

$$\begin{aligned}x &= y + 5 \\x - x &= y - x + 5 \\0 &= y - x + 5 \\0 - 5 &= y - x + 5 - 5 \\-5 &= y - x\end{aligned}$$

Now you can find the value of  $(y - x)^3$  by putting  $-5$  in for  $y - x$ . That gets you  $-125$ .

12. J

In geometry you learned that the smallest side of a triangle is opposite the smallest angle, the middle side opposite the middle angle and the largest side opposite the largest angle. That tells you that  $BC$  is the smallest side,  $AB$  is the middle side, and  $AC$  is the largest side. Therefore,  $BC < AB < AC$ .

13. D

If  $x = -3$  is a solution to the equation, then it must also be true that  $(x + 3)$  is a factor of the polynomial. Since  $x = -3$  is the *only* solution,  $(x + 3)$  must appear as a factor twice. You can write the polynomial as the product of these two factors and then multiply (FOIL) to transform the polynomial into its  $x^2 + bx + c$  form.

$$\begin{aligned}(x + 3)(x + 3) &= 0 \\x^2 + 3x + 3x + 9 &= 0 \\x^2 + 6x + 9 &= 0\end{aligned}$$

The value of  $b$  is 6.

14. G

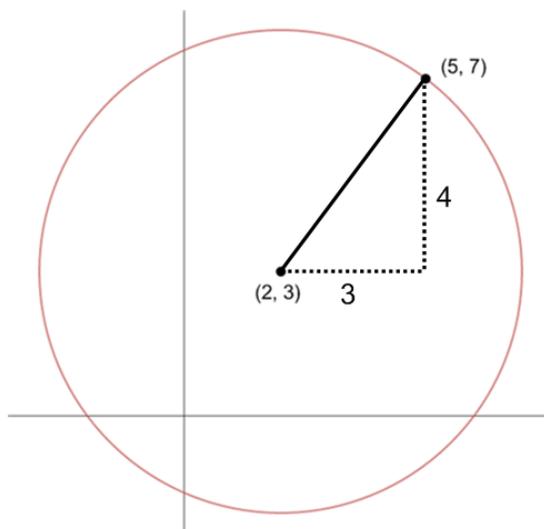
To find the area of the circle, you need to know its radius. Because you know the coordinates of the center of the circle and the coordinates of a point on the circle, you can use the Distance Formula to find the length of the radius.

$$\begin{aligned}d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\d &= \sqrt{(5 - 2)^2 + (7 - 3)^2} \\d &= \sqrt{3^2 + 4^2} \\d &= \sqrt{9 + 16} \\d &= \sqrt{25} \\d &= 5\end{aligned}$$

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The radius of the circle is 5.

You can also find the radius of the circle using the Pythagorean Theorem. First draw in a right triangle with the radius as the hypotenuse.



$$\begin{aligned}a^2 + b^2 &= c^2 \\3^2 + 4^2 &= c^2 \\9 + 16 &= c^2 \\25 &= c^2 \\5 &= c\end{aligned}$$

You can now find the area of the circle using the formula  $A = \pi r^2$ . The area of this circle is  $\pi(5)^2 = 25\pi$ .

15. C

You can do this problem using a process called dimensional analysis (or “label cancelling”) that you might be familiar with from Chemistry class.

$$\frac{1.49 \times 10^3 \text{ meters}}{1 \text{ second}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} \times$$

$$5 \text{ minutes} = 447,000 \text{ meters}$$

$$= 4.47 \times 10^5 \text{ meters}$$

16. K

Problems about averages are often best solved by looking at the *total*. If Brendan has 6 tests and he wants to have an average of 90, his tests scores need to add up to  $6 \times 90 = 540$ . You can next find out what the total of the first five test scores is.

$$\text{First 3 tests: } 89 \times 3 = 267$$

$$\text{Next 2 tests: } 87 \times 2 = 174$$

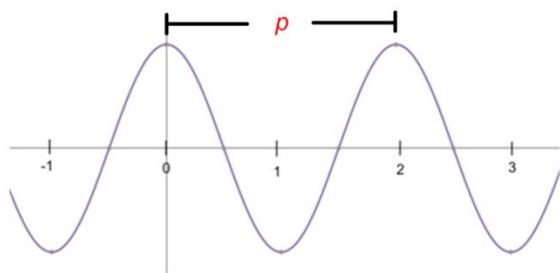
$$\text{Total on 5 tests: } 267 + 174 = 441$$

If the total on the first 5 tests was 441 and Brendan needs a total of 540 on the 6 tests, he needs to earn a score of  $540 - 441 = 99$  on the final test.

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17. B

One way of thinking about the period of a function is to realize that it is the horizontal distance that it takes the function to complete one full “pattern” (you can talk about the period of a function only when a portion of that function repeats itself exactly over and over again).



The period of this function is 2.

18. H

You can use the formula for the sum of an arithmetic sequence to find the first term.

$$S = \frac{n(a_1 + a_n)}{2}$$

$$78 = \frac{6(a_1 + 23)}{2}$$

$$78 = 3(a_1 + 23)$$

$$26 = a_1 + 23$$

$$3 = a_1$$

Now you know the sequence looks like this:

$$3 \quad \_ \quad \_ \quad \_ \quad \_ \quad 23$$

To get from the first term to the sixth you have five equivalent “jumps.” The values increased from 3 to 23 in those jumps, so each one must be  $20 \div 5 = 4$ . Therefore, the second term must be  $3 + 4 = 7$ .

19. D

For this problem, you need to use an identity from trigonometry:

$$\cos^2\theta + \sin^2\theta = 1$$

Start the problem by expanding and then simplifying the expression.

$$(3\cos\theta - 3\sin\theta)^2$$

$$(3\cos\theta - 3\sin\theta)(3\cos\theta - 3\sin\theta)$$

$$9\cos^2\theta - 9\cos\theta\sin\theta - 9\cos\theta\sin\theta + 9\sin^2\theta$$

$$9\cos^2\theta - 18\cos\theta\sin\theta + 9\sin^2\theta$$

This isn’t one of the answer choices, so you need to think about how you might simplify the expression. This is where the identity comes in.

$$9\cos^2\theta + 9\sin^2\theta - 18\cos\theta\sin\theta$$

$$9(\cos^2\theta + \sin^2\theta) - 18\cos\theta\sin\theta$$

$$9(1) - 18\cos\theta\sin\theta$$

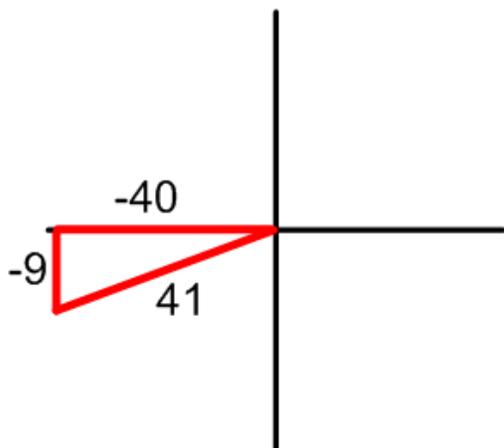
$$9 - 18\cos\theta\sin\theta$$

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20. G

Draw the triangle, labeling each side while paying attention to both its size and sign. You can find the length of the hypotenuse using the Pythagorean Theorem.

$$\begin{aligned}a^2 + b^2 &= c^2 \\9^2 + 40^2 &= c^2 \\81 + 1600 &= c^2 \\1681 &= c^2 \\41 &= c\end{aligned}$$



The sine of the angle is the ratio of the length of the opposite side to the length of the hypotenuse, so in this case  $\sin\theta = -9/41$ .

## Want More?

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<http://blog.cardinallec.com/tag/act/>

There you'll find posts that review the topics that are tested on the ACT as well as a lot more practice problems.

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