Solutions – Quadratics & Parabolas

1) C

It might be more efficient on this problem to simply test the answer choices until you find the one that contains the solutions. However, if you did the math, it would look like this:

$$(x + 1)(x – 2) = 10$$

$$x^2 – 2x + 1x – 2 = 10$$

$$x^2 – 1x – 2 = 10$$

$$x^2 – 1x – 12 = 0$$

$$(x – 4)(x + 3) = 0$$

$$x = 4; x = -3$$

2) A

This problem simply tests your ability to factor a quadratic trinomial in order to find the solutions to the equation.

$$3n^2 + 21n + 30 = 0$$

$$3(n^2 + 7n + 10) = 0$$

$$3(n + 2)(n + 5) = 0$$

$$n = -2; n = -5$$

The sum of the solutions is -2 + (-5) = -7.

(Note: You could also have gotten the two solutions using the Quadratic Formula.)

3) B

You would probably start this problem by seeing if you could factor the trinomial on the left side of the equation. When you realize that you can't, the next option would be Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{6 - 4(1)(4)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{20}}{2}$$

$$x = \frac{-6 \pm 2\sqrt{5}}{2}$$

$$x = -3 \pm \sqrt{5}$$

If you had forgotten how to use the Quadratic Formula, you could have graphed $y = x^2 + 6x + 4$ on your graphing calculator. Then find the zeroes of the function (the points where the graph crosses the *x*-axis). Compare the decimals that you get with the decimal equivalents of the answer choices until you find the correct solutions.

4) B

All of the answer choices, except for Choice C, are equivalent to the original equation. But the question asks us to find the one that contains the coordinates of the vertex. So how do we find the vertex of a quadratic in $y = ax^2 + bx + c$ form?

You'll recall that a parabola can be "split in half" by drawing a vertical line, called the axis of symmetry, down the middle of the graph. The equation of this line can be found using the following equation:

$$x = \frac{-b}{2a} = \frac{6}{2(1)} = \frac{6}{2} = 3$$

So the axis of symmetry is the vertical line $x = 3$. This line passes through the vertex, so that the x-coordinate of the vertex must also be 3. To get the y-coordinate of the vertex, simply run 3 through the function. Doing this gets you $y = 3^2 - 6(3) + 10 = 1$. The vertex is (3, 1).

5) B

The zeroes of the function are $x = 3$ and $x = -5$; these are the points where the graph crosses the x-axis. Because of the symmetry properties of parabolas, you know that the axis of symmetry must be halfway between these two values at $x = -1$. The vertex, therefore, must have an x-coordinate of -1. Because only one of the answer choices has this x-coordinate, that must be the answer. However, if you needed the y-coordinate (in terms of a), you could put -1 into the equation in place of x.

6) D

This question tests your knowledge of the symmetry properties of parabolas. If the maximum value of the function is $g(4)$, then the axis of symmetry must be the vertical line $x = 4$. Points that are the same horizontal distance away from this axis must have the same y-coordinate. Additionally, points that are *not* the same horizontal distance from the axis *cannot* have the same y-coordinate. The points (-1, 10) and (9, 10) are both five horizontal units from the axis of symmetry and they have the same y-coordinate. That's your answer.

7) 11

Let's consider what this trinomial must look like factored

$$6x^2 + bx - 10$$

$$(2x + 5)(\_\_\_x + \_\_\_)$$

In order to get a $6x^2$ term, the first blank above must be a 3, and to get a -10 term, the second blank must be a -2. So now we have:

$$6x^2 + bx - 10$$

$$(2x + 5)(3x - 2)$$

$$6x^2 - 4x + 15x - 10$$

$$6x^2 + 11x - 10$$

8) D

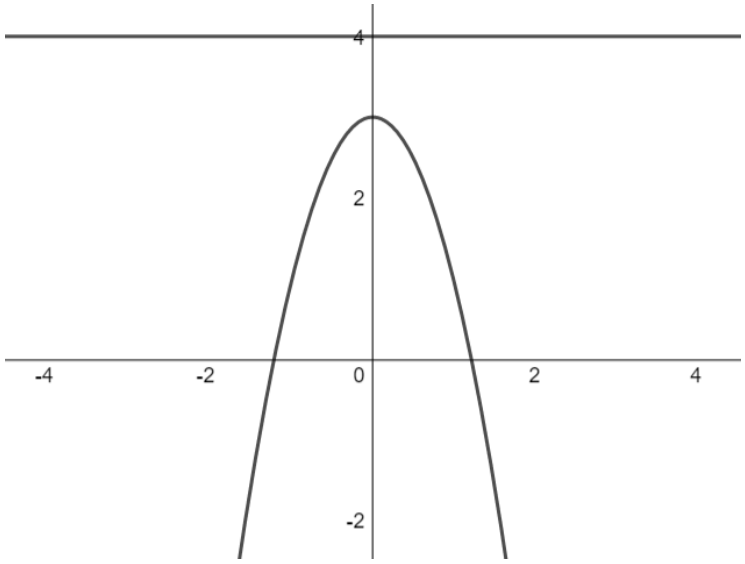
The first equation is a horizontal line, and the second equation is a parabola. If the system has *no solution*, the two graphs don't intersect. This can happen if one of two cases occurs:

- The parabola opens *up* and the vertex is *above* the line
- The parabola opens *down* and the vertex is *below* the line

Because there is no x term in the parabola, its vertex is on the y-axis and will occur at the point with coordinates $(0, b)$

If you look at the answer choices carefully, you'll see that Choice D would give you a parabola that opens down (because the value of a is negative) and that has a vertex (or maximum) at the point (0, 3). That parabola won't intersect the line.

If you were not sure about these characteristics of parabolas, you could have tried each of the answer choices on your graphing calculator. Choice D would look like this:



9) .925

The object hits the ground when the height is 0, so put 0 into the equation for $h(t)$ and solve. You could do this by graphing it on your calculator and seeing where the parabola intersects the horizontal axis. Note that there are two such points, but time has to be positive, so that's the one that you want.

To do the problem by hand, you'll need to use the Quadratic Formula.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-4 \pm \sqrt{4^2 - 4(-16)(10)}}{2(-16)}$$

$$t = \frac{-4 \pm \sqrt{656}}{-32}$$

$$t \approx 0.925$$

10) 3/4 or .75

The zeroes of the function are $x = 3$ and $x = 7$, so the graph crosses the x-axis at (3, 0) and (7, 0). The axis of symmetry is halfway between these points; it is the vertical line $x = 5$. Because the vertex is on the axis of symmetry and we are told that the minimum value of the function is -3, we know that the coordinates of the vertex are (5, -3).

You can now plug that point into the equation and solve for a.

$$f(x) = a(x - 3)(x - 7)$$

$$-3 = a(5 - 3)(5 - 7)$$

$$-3 = a(2)(-2)$$

$$-3 = a(-4)$$

$$3/4 = a$$