Solutions – Completing the Square

1. C

You can solve this problem by using "complete the square" to rewrite the given equation in the vertex form of the parabola: $f(x) = (x - h)^2 + k$. In this form (h, k) is the vertex of the parabola.

$$f(x) = x^2 - 12x + 40$$
$$f(x) = x^2 - 12x + \_\_\_ + 40$$
$$f(x) = x^2 - 12x + 36 + 40 - 36$$
$$f(x) = (x^2 - 12x + 36) + 4$$
$$f(x) = (x - 6)(x - 6) + 4$$
$$f(x) = (x - 6)^2 + 4$$

The number that went in the blank to complete the square was found by cutting -12 in half and then squaring the result. Once the equation of the function is in vertex form, you can see that the vertex is (6, 4).

Note: You could also have found the vertex by finding the axis of symmetry using the equation $x = -b/2a$. This will give you the x-coordinate of the vertex, and you can run this number through the function to get the y-coordinate.

2. B

The graph of the function is a parabola that opens upward (you know this because the coefficient of the "squared" term is positive). The minimum value of the function is, therefore, the y-coordinate of the vertex. You could find the coordinates of the vertex using either of the two methods described in the first problem above. Here's the work if you decide to complete the square.

$$f(x) = x^2 + 8x + 18$$
$$f(x) = x^2 + 8x + \_\_\_ + 18$$
$$f(x) = x^2 + 8x + 16 + 18 - 16$$
$$f(x) = (x^2 + 8x + 16) + 2$$
$$f(x) = (x + 4)(x + 4) + 2$$
$$f(x) = (x + 4)^2 + 2$$

The vertex of the parabola is (-4, 2) and the minimum value of the function is 2.

3. D

To find the value of c, you would simply cut the coefficient of x, which in this case is -10, in half and then square the result. So $c = (-10/2)^2 = (-5)^2 = 25$.

4. B

The equation of a circle can be written in the form $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center of the circle and r is the radius. You can use "complete the square" twice, once on each variable, to rewrite the given equation in this form.

$$x^2 + y^2 - 2x + 10y + 10 = 0$$
$$x^2 - 2x + \_\_\_ + y^2 + 10y + \_\_\_ = -10$$
$$x^2 - 2x + 1 + y^2 + 10y + 25 = -10 + 1 + 25$$
$$(x^2 - 2x + 1) + (y^2 + 10y + 25) = 16$$
$$(x - 1)(x - 1) + (y + 5)(y + 5) = 16$$
$$(x - 1)^2 + (y + 5)^2 = 4^2$$

The center of the circle is at the point (1, -5) and the radius is 4.

5. C

Use the same technique you used in the previous problem.

$$x^2 + y^2 - 12x + 6y - 19 = 0$$
$$x^2 - 12x + \_\_\_ + y^2 + 6y + \_\_\_ = 19$$
$$x^2 - 12x + 36 + y^2 + 6y + 9 = 19 + 36 + 9$$
$$(x^2 - 12x + 36) + (y^2 + 6y + 9) = 64$$
$$(x - 6)(x - 6) + (y + 3)(y + 3) = 64$$
$$(x - 6)^2 + (y + 3)^2 = 8^2$$

The center of the circle is at the point (6, -3) and the radius is 8.