



ACT Math
Diagnostic
Test #1

Art Cockerham

ACT Math Diagnostic Test #1

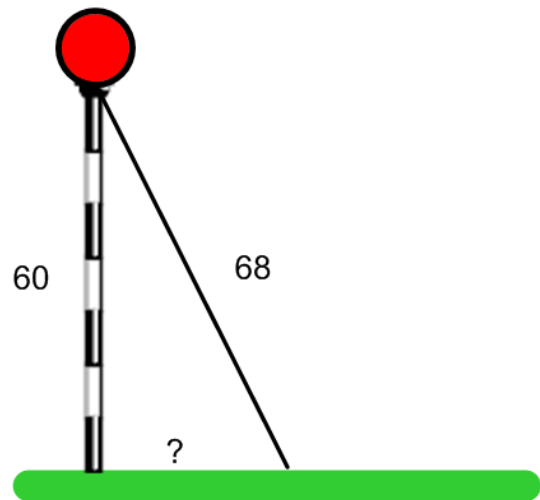
1) On level ground, a flagpole casts a shadow that is 8 feet long. At the same time, a 5-foot tall person casts a shadow that is 2 feet long. How many feet tall is the flagpole?

- A. 3.2
- B. 4
- C. 12
- D. 16
- E. 20

2) What is the value of x in the equation $3(x - 4) + 2x = 18$?

- F. 1.2
- G. 2
- H. 4
- J. 6
- K. 8

3) A 60-foot tall beacon tower is supported by a wire that is 68 feet long, as shown below. How many feet from the base of the tower is the wire attached to the ground?



- A. 8
- B. 12
- C. 24
- D. 32
- E. 40

ACT Math Diagnostic Test #1

4. For all values of x , $(5x + 3)^2 = ?$

- F. $10x + 6$
- G. $10x^2 + 6$
- H. $25x^2 + 9$
- J. $25x^2 + 15x + 9$
- K. $25x^2 + 30x + 9$

5. The line $y = \frac{2}{3}x - 4$ graphed in the standard (x, y) coordinate plane crosses the x -axis at the point (a, b) . What is the value of a , the x -coordinate of that point?

- A. -6
- B. 0
- C. 4
- D. 6
- E. 9

6. John has saved \$120 and he is planning to add \$10 per week to his account. Sarah has saved \$90 and she is planning to add \$15 per week to her account. If both John and Sarah are able to save at this constant rate, in how many weeks will they have the same amount of money?

- F. 2
- G. 3
- H. 4
- J. 6
- K. 9

7. What is the slope of any line that is perpendicular to $8x - 4y = 20$?

- A. -2
- B. $-\frac{1}{2}$
- C. $-\frac{1}{4}$
- D. 2
- E. 8

ACT Math Diagnostic Test #1

8. When you subtract 3 times a number n from 12, the result is positive. Which of the following gives the possible value(s) for n ?

- F. 0 only
- G. 4 only
- H. All $n > 0$
- J. All $n > 4$
- K. All $n < 4$

9. What is the the x -coordinate of the point in the standard (x, y) coordinate plane at which the lines $y = -4x + 7$ and $y = 12x - 1$ intersect?

- A. $-1/2$
- B. $1/2$
- C. 1
- D. 2
- E. 5

10. If $|x - 6| = 10$, then $x = ?$

- F. 4 only
- G. 16 only
- H. {4, 16}
- J. {-6, 10}
- K. {-4, 16}

11. In an arithmetic sequence, each term after the first term is found by adding the same number to the preceding term. In an arithmetic sequence, the 4th term is 22 and the 6th term is 34. What is the first term?

- A. -2
- B. 4
- C. 6
- D. 10
- E. 16

ACT Math Diagnostic Test #1

12. A circle in the standard (x, y) coordinate plane has a diameter with endpoints at the points $(-2, 7)$ and $(6, 7)$. What is the area of the circle?

- F. 4π
- G. 8π
- H. 9π
- J. 16π
- K. 64π

13. In the right triangle below, what is the value of $\sin B$?

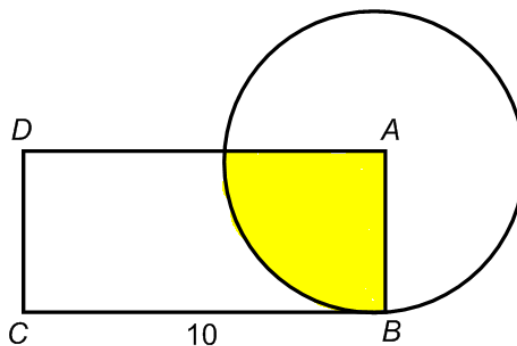


- A. $5/12$
- B. $5/13$
- C. $12/13$
- D. $13/12$
- E. $13/5$

14. If 120% of a certain number is 960, what is 25% of the number?

- F. 192
- G. 196
- H. 200
- J. 240
- K. 600

15. In the figure below, segment CB is tangent to the circle at Point B and has a length of 10 units. The circle has its center at Point A . If the perimeter of rectangle $ABCD$ is 28, what is the area, in square units, of the shaded region?



- A. 2π
- B. 4π
- C. 8π
- D. 10π
- E. 16π

ACT Math Diagnostic Test #1

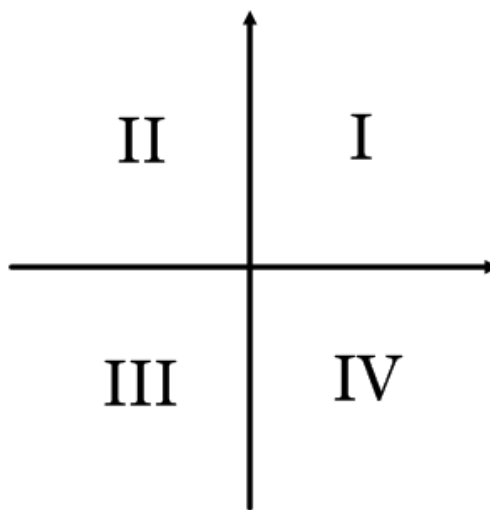
16. Which of the following expressions, if any, are equal for all real numbers x ?

- I. $-x^2$
 - II. $(-x)^2$
 - III. $\sqrt{x^4}$
- F. I and II only
G. I and III only
H. II and III only
J. I, II, and III
K. None of the expressions are equivalent for all real numbers x .

17. If $2x^2 + y$ is an even integer, which of the following could be true about x and y ?

- I. x is odd; y is odd
 - II. x is even; y is odd
 - III. x is odd; y is even
- A. I only
B. II only
C. III only
D. I and II
E. I, II and III

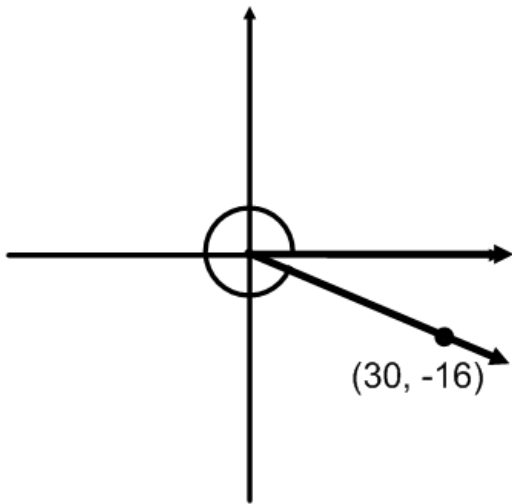
18. The x -axis and y -axis together divide the standard (x, y) coordinate plane into four quadrants as shown in the diagram below. If the circle represented by the equation $(x - 2)^2 + (y + 6)^2 = 16$ is graphed in the standard xy -coordinate plane, in what quadrant(s) will the circle lie?



- F. I only
G. IV only
H. I and II
J. I and IV
K. III and IV

ACT Math Diagnostic Test #1

19. In the standard (x, y) coordinate plane below, an angle is shown whose vertex is the origin. One side of this angle θ passes through the point $(30, -16)$, and the other side of the angle is on the positive x -axis. What is the cosine of θ ?



- A. $-15/17$
- B. $-8/17$
- C. $8/15$
- D. $8/17$
- E. $15/17$

20. If $\log_b x = m$ and $\log_b y = n$, then $\log_b(xy^3) = ?$

- F. $3mn$
- G. $6mn$
- H. $3(m + n)$
- J. $m + 3n$
- K. mn^3

ACT Math Diagnostic Test #1

Answer Key

| Answer | EA | AG | GT | Topic(s) |
|--------|----|----|----|----------------------------------------------------|
| 1. E | X | | | Proportions |
| 2. J | X | | | Solving Equations |
| 3. D | | | X | Pythagorean Theorem |
| 4. K | X | | | Multiplying Polynomials |
| 5. D | | X | | Graphs of Linear Equations |
| 6. J | X | | | Writing & Solving Equations |
| 7. B | | X | | Equations of Lines; Parallel & Perpendicular Lines |
| 8. K | | X | | Writing & Solving Inequalities |
| 9. B | | X | | Graphs of Linear Equations; Solving Equations |
| 10. K | X | | | Solving Absolute Value Equations |
| 11. B | | X | | Sequences |
| 12. J | | | X | Circles |
| 13. C | | | X | Right Triangle Trigonometry (SOH-CAH-TOA) |
| 14. H | X | | | Percents; Proportions |
| 15. B | | | X | Perimeter; Area of Circles |
| 16. H | X | | | Exponents; Square Roots |
| 17. C | X | | | Properties of Numbers |
| 18. K | | X | | Circles; Quadrants |
| 19. E | | | X | Right Triangle Trigonometry (SOH-CAH-TOA) |
| 20. J | | X | | Logarithms |

EA = Pre-Algebra & Elementary Algebra

AG = Intermediate Algebra & Coordinate Geometry

GT = Plane Geometry & Trigonometry

ACT Math Diagnostic Test #1

Solutions

1. E

This problem can be solved by setting up a proportion and then solving.

$$\frac{x}{8} = \frac{5}{2}$$

Now solve the equation using the fact that the cross products of a proportion are equal.

$$2x = 8(5)$$

$$2x = 40$$

$$x = 20$$

2. J

Solve the equation using the distributive rule and combining like terms.

$$3(x - 4) + 2x = 18$$

$$3x - 12 + 2x = 18$$

$$5x - 12 = 18$$

$$5x - 12 + 12 = 18 + 12$$

$$5x = 30$$

$$5x/5 = 30/5$$

$$x = 6$$

3. D

The tower, wire and ground form a right triangle. You can find the missing distance using the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$

$$a^2 + 60^2 = 68^2$$

$$a^2 + 3600 = 4624$$

$$a^2 + 3600 - 3600 = 4624 - 3600$$

$$a^2 = 1024$$

$$a = \sqrt{1024}$$

$$a = 32$$

4. K

Recognize that when you square a binomial, this is really a "FOIL problem in disguise."

$$(5x + 3)^2$$

$$(5x + 3)(5x + 3)$$

$$25x^2 + 15x + 15x + 9$$

$$25x^2 + 30x + 9$$

ACT Math Diagnostic Test #1

5. D

When a line crosses the x -axis, the point where it crosses has a y -coordinate of zero. You can replace the y in the equation with 0 and solve for x .

$$\begin{aligned}y &= \frac{2}{3}x - 4 \\0 &= \frac{2}{3}x - 4 \\0 + 4 &= \frac{2}{3}x - 4 + 4 \\4 &= \frac{2}{3}x \\(\frac{3}{2})(4) &= (\frac{3}{2})(\frac{2}{3}x) \\6 &= x\end{aligned}$$

Alternatively, you could graph the line on your graphing calculator and look at where the line crosses the x -axis.

6. J

You can write expressions for how much money each of them will have after w weeks.

John: $120 + 10w$

Sarah: $90 + 15w$

Now write and solve an equation to determine when the amount of money they each have is the same.

$$\begin{aligned}120 + 10w &= 90 + 15w \\120 + 10w - 10w &= 90 + 15w - 10w \\120 &= 90 + 5w \\120 - 90 &= 90 - 90 + 5w \\30 &= 5w \\30/5 &= 5w/5 \\6 &= w\end{aligned}$$

7. B

Begin by re-arranging the line into slope-intercept form ($y = mx + b$).

$$\begin{aligned}8x - 4y &= 20 \\-4y &= -8x + 20 \\-4y/-4 &= -8x/-4 + 20/-4 \\y &= 2x - 5\end{aligned}$$

The slope of this line is 2. Recall that perpendicular lines have slopes that are *opposite reciprocals*, so a line perpendicular to this one would have a slope of $-1/2$.

8. K

Begin by writing an inequality and then solve it.

$$\begin{aligned}12 - 3n &> 0 \\12 - 12 - 3n &> 0 - 12 \\-3n &> -12 \\-3n/-3 &< -12/-3 \\n &< 4\end{aligned}$$

In the last step, it's important you recall that if you *multiply* or *divide* both sides of an inequality by a negative number, you need to switch the direction of the inequality symbol.

ACT Math Diagnostic Test #1

9. B

Because both $-4x + 7$ and $12x - 1$ are equal to y , you can write an equation that sets them equal to each other. Then solve for x .

$$\begin{aligned} -4x + 7 &= 12x - 1 \\ -4x + 4x + 7 &= 12x + 4x - 1 \\ 7 &= 16x - 1 \\ 7 + 1 &= 16x - 1 + 1 \\ 8 &= 16x \\ 8/16 &= 16x/16 \\ 8/16 &= x \\ 1/2 &= x \end{aligned}$$

10. K

You can solve an absolute value equation by writing two equations.

$$\begin{aligned} x - 6 &= 10 \\ x - 6 + 6 &= 10 + 6 \\ x &= 16 \\ \text{and} \\ x - 6 &= -10 \\ x - 6 + 6 &= -10 + 6 \\ x &= -4 \end{aligned}$$

You also could have tried each of the answer choices in the equation. However, be careful with Choice G. While 16 is a solution to the equation, it might not be the *only* solution. After you figure out that 16 does indeed work, you need to try any of the other choices that also include 16.

11. B

Begin with six blanks on your paper and fill in what you know.

$$\underline{\quad} \quad \underline{\quad} \quad \underline{\quad} \quad 22 \quad \underline{\quad} \quad 34$$

From the 4th term to the 6th term, the pattern increased by 12 units in the span of 2 “jumps.” Therefore, each jump must be 6 units. That’s the common difference that is added to create the next term in the sequence. Now go back and fill in the blanks in such a way that the difference between each term is 6.

$$4 \quad 10 \quad 16 \quad 22 \quad 28 \quad 34$$

The first term is 4.

12. J

The diameter has endpoints at $(-2, 7)$ and $(6, 7)$. Therefore, its length is 8 units and the radius of the circle is 4 units. You can use the area formula for a circle to answer the question.

$$\begin{aligned} A &= \pi r^2 \\ A &= \pi(4)^2 \\ A &= 16\pi \end{aligned}$$

ACT Math Diagnostic Test #1

13. C

Begin by using the Pythagorean Theorem to find the length of the hypotenuse.

$$\begin{aligned}a^2 + b^2 &= c^2 \\5^2 + 12^2 &= c^2 \\25 + 144 &= c^2 \\169 &= c^2 \\\sqrt{169} &= c \\13 &= c\end{aligned}$$

From SOH-CAH-TOA, we know that the sine of an angle is the ratio of the opposite side to the hypotenuse. Therefore, $\sin B = 12/13$.

14. H

Like many problems on the ACT, this one can be solved in a couple of different ways. One possibility is that you start by writing an equation to find the number.

$$\begin{aligned}1.2n &= 960 \\1.2n / 1.2 &= 960 / 1.2 \\n &= 800\end{aligned}$$

Next find 25% of 800, which is 200.

Or you could have written and solved a proportion.

$$\begin{aligned}\frac{120}{960} &= \frac{25}{n} \\120n &= 25(960) \\120n &= 24,000 \\n &= 200\end{aligned}$$

15. B

You should first use the perimeter formula for a rectangle to find the width of the rectangle (which is also the radius of the circle).

$$\begin{aligned}2L + 2W &= P \\2(10) + 2W &= 28 \\20 + 2W &= 28 \\2W &= 8 \\W &= 4\end{aligned}$$

Now you can use the fact that the radius of the circle is 4 to find the area of the circle.

$$\begin{aligned}A &= \pi r^2 \\A &= \pi(4)^2 \\A &= 16\pi\end{aligned}$$

The shaded portion of the circle is a quarter circle because $\angle CAB$ is an angle of the rectangle and, therefore, 90 degrees. So the shaded region is equal to $1/4(16\pi) = 4\pi$.

16. H

The first choice is $-x^2$ while the other two are equivalent to x^2 . To verify this, you could try each option with a number. If you were to use -3, it would look like this:

$$\begin{aligned}\text{I. } -x^2 &= -(-3)^2 = -9 \\ \text{II. } (-x)^2 &= (3)^2 = 9 \\ \text{III. } \sqrt{(-3)^4} &= \sqrt{81} = 9\end{aligned}$$

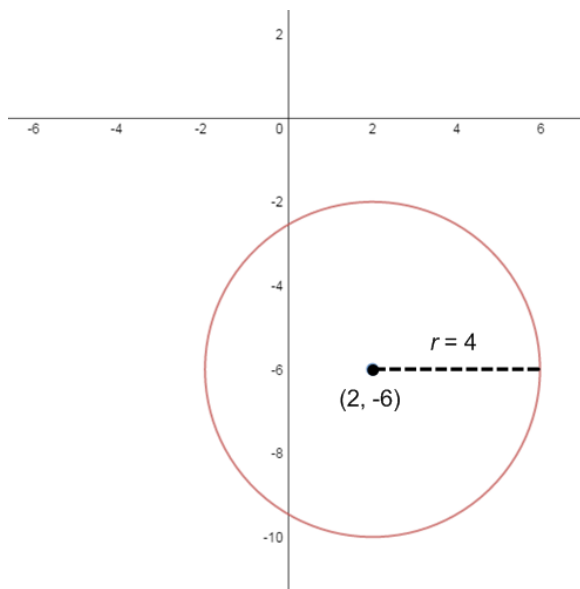
ACT Math Diagnostic Test #1

17. C

The key to this problem is to recognize that $2x^2$ is even no matter what the value of x is (because you are multiplying the x^2 by 2). Therefore, the only way to make the sum even is to make y even, because an even added to an even will be even, while an even added to an odd will be odd.

18. K

The equation of a circle is given by the equation $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center of the circle and r is the radius of the circle. The equation of our circle is $(x - 2)^2 + (y + 6)^2 = 16$, which can be thought of as $(x - 2)^2 + (y - (-6))^2 = 4^2$. This tells you that the center of the circle is $(2, -6)$ and that the radius is 4. You can now sketch the circle in the standard xy -coordinate plane.

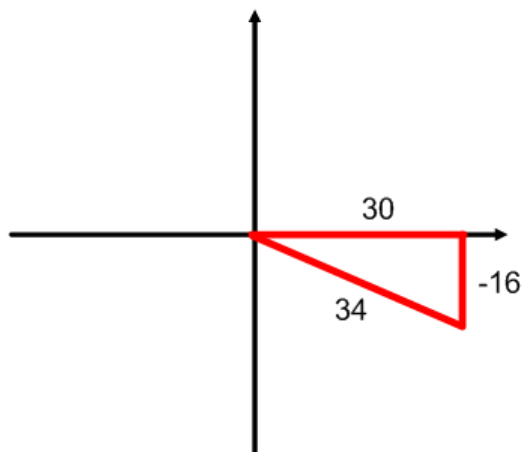


Doing so will show you that the circle lies in Quadrant III and Quadrant IV.

19. E

Begin by sketching the relevant triangle in the 4th quadrant. Then use the Pythagorean Theorem to find the missing length of the hypotenuse. Be sure to pay attention to signs as you label the sides of the triangle.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 30^2 + 16^2 &= c^2 \\ 900 + 256 &= c^2 \\ 1156 &= c^2 \\ \sqrt{1156} &= c \\ 34 &= c \end{aligned}$$



You know from SOH-CAH-TOA that cosine is the ratio of the adjacent side to the hypotenuse, so $\cos \theta = 30/34 = 15/17$.

ACT Math Diagnostic Test #1

20. J

In order to get this problem correct, you must be familiar with your log rules, which are summarized in the table below.

| Name of Rule | Rule |
|--------------|-------------------------------|
| Product | $\log(AB) = \log A + \log B$ |
| Quotient | $\log(A/B) = \log A - \log B$ |
| Power | $\log A^B = B \cdot \log A$ |

For this problem, we'll need the Product Rule and the Power Rule to simplify the expression $\log_b xy^3$.

$$\begin{aligned}\log_b xy^3 \\ \log_b x + \log_b y^3 \\ \log_b x + 3\log_b y \\ m + 3n\end{aligned}$$

Recall that in the problem it was given that $\log_b x = m$ and $\log_b y = n$.

Want More?

If you found this diagnostic test helpful and you'd like to learn more about the ACT Math test, you can head on over to

<http://blog.cardinalec.com/tag/act/>

There you'll find posts that review the topics that are tested on the ACT as well as a lot more practice problems.

You can also follow my college admissions and test preparation firm, Cardinal Educational Consulting, on social media.



[Twitter](#)



[Facebook](#)

If you have questions, please feel free to send an email.



<mailto:info@cardinalec.com>